An Ethics of Teaching and Learning Mathematics

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INTRODUCTION

"Why are we learning this?" Students ask mathematics teachers this question, sometimes sincerely and sometimes less so, so often that many teachers have developed stock responses. Common responses include the authoritarian "because I said so," the utilitarian "because it will be useful," the conspiratorial "because it's fun," and the existentially honest "I don't know." In this paper, I consider whether there is an ethical reason to teach and learn mathematics, because answering this question can inform how, if at all, mathematics education should be pursued. To be clear, my question is not about an ethical reason for compulsory education with mathematics as one part, nor is it about the ethics of teaching mathematics.¹ Kurt Stemhagen has argued that mathematics is an "ethics-laden enterprise,"² and this paper builds on his claim by investigating the possible ethical value of mathematics itself: of mathematics as an intrinsically ethical endeavor. First, I identify limitations of reasons commonly offered for teaching and learning mathematics. Next, I argue that ethical reasoning requires expanding what is considered "practical" beyond individual and economic needs to include the well-being of humanity as a collective. By looking at what mathematics is and how it functions in human interaction, I examine the purposes and possibilities of mathematical ideas and practices. I conclude by suggesting one reason that the teaching and learning of mathematics may not only be ethically permissible, but also ethically necessary.

WHY TEACH AND LEARN MATHEMATICS?

The reasons most often given to students, and the reasons pervasive in education policy and research, are economically functionalist: students need to learn mathematics to go to the grocery store and file taxes; to improve their chances of being accepted to college and avoiding remedial courses; and to pursue prestigious jobs in science and engineering. Prioritizing students' future earning potentials and financial transactions as the purpose of mathematics education, however, centers money as their primary concern. Promoting college and career readiness as the goal of mathematics education, similarly, foregrounds the role of education in sorting people based on their presumed intellectual capability, an idea that has historically served to legitimize and uphold racial hierarchy in the United States.³ These practical explanations for learning mathematics might make sense in a sociopolitical context in which the worth of a person is premised on their contributions—or potential for contribution—to the economy, yet this is a reductive approach to human life.⁴

Sometimes, teaching and learning mathematics is justified through the broadly applicable skills that students might develop: even if they never factor a quadratic again, students can practice problem-solving. Alternately, learning mathematics is portrayed as an opportunity for self-actualization; a growing body of research sees classrooms as sites for disrupting assumptions about who can participate in mathematics and who is competent in mathematics, expanding students' understanding of who mathematicians can and should be.⁵ Such reasons raise the question of why these skills are tied to mathematics; why not encourage students to develop critical thinking or confidence through debate or sport or other activities? Occasionally, aesthetic warrants are offered. There is beauty and joy to be discovered in mathematics, particularly if students are able to engage in the dialogic, creative aspects of mathematical work—"real math"—rather than what is called "school math," the procedural, formulaic mathematics that is taught, learned, and valued in most K-12 schools in the United States.⁶

These practical, skills-based, and aesthetic claims, however, foreground benefits for individual students and for the economy. Unless we believe that individual fulfillment and economic growth are sufficient markers of a thriving society, these reasons fail to account for the pragmatics of collective coexistence on a planet with seven billion humans and countless more living and non-living beings. This seems like a gross oversight, given that the teaching and learning of mathematics carries collective consequences. For example, the experience of learning school math leaves many students feeling alienated, helpless, and even incompetent.⁷ Applications of mathematics have brought us computers and nuclear weapons. They have allowed us to make life-prolonging medical breakthroughs, and also to justify the purported superiority of people marked as White during the eugenics movement. Mathematical applications become algorithms that meter traffic on busy interstates, and also disproportionately target people of color for predatory policing. Moving from saying that the teaching and learning of mathematics is ethically-laden to saying that it is ethical, then, requires accounting for both individual and collective well-being.

WHAT WOULD AN ETHICAL REASON BE?

I start from the premise that the world is not of our making. Not only are we shaped and constrained by histories and norms and structures that far predated us, but also these histories and norms and structures are fraught with power relations that create and maintain conditions of material poverty and denial of dignity and humanity for many people—especially the Black, the female, those from the global South, those from the Orient, the queer, and the poor.⁸ One might think, then, that an ethical position would be one that overturns such power relations. This is the position taken by critical mathematics educators, who acknowledge the role that mathematics and mathematics educators encourage students to use mathematical analyses to reveal and challenge inequitable power relations; their approach is also called social justice mathematics.¹⁰ Critical mathematics complicates the traditional teaching and learning of mathematics by adding socially just and democratic aims to the socially reproductive and economic aims that might otherwise prevail.

One might argue, however, that critical mathematics uses the master's tools to attempt to dismantle the master's house.¹¹ It leaves unquestioned the substance and the primacy of school math, and also the "formatting power of mathematics,"¹² or the power of those who know mathematics to organize the

reality of and make decisions—possibly in coercive ways—on behalf of those who do not. In other words, even if used to combat disproportionate incarceration rates or housing discrimination, school math maintains a privileged place as a tool that empowers those who wield it, and those who know school math remain more powerful than those who do not. Furthermore, it seems unlikely that, even with critical mathematics education for all, existing power relations could simply be replaced by a system that is more equitable for all people at all times. The complexity of intersecting identities means that no identity is always oppressed or always free in all contexts, and so it would be impossible to design a system in which everyone is universally free from being on the receiving end of inequitable power relations.¹³ Finally, the ideal of individual freedom as an important outcome is itself tied to a particular "regime of truth" that privileges individuals as distinct from and more deserving of well-being or liberation than a collective.¹⁴

If not simply seeking freedom from the current system, what does it mean to act ethically in light of histories and norms and structures that shape and constrain, and that do so in oppressive ways? "We are not deterministically decided by norms,"¹⁵ Judith Butler offers, but rather, norms form a backdrop in relation to which our actions take place. The perpetuation of existing power relations and existing conditions of oppression relies on people's action to re-produce—to continue to be normal—and this reliance "provides the conditions for subversion and critique."¹⁶ Therefore, acting ethically requires "becoming knowing about the ways in which our actions are taken up by the already-constituted social world and what consequences will follow from our acting in certain ways."¹⁷ In other words, constant self-questioning, as opposed to universal rules of behavior, is key to ethical action.

Self-questioning can result in action, or it can result in action by inaction, a product of relativistic reasoning that suggests everything is equally ethical. As much as there is rarely (dare I say never) a single "right" action, an ethics according to what I have described above requires, for one, choosing actions that avoid coercive power. It would emphasize the pragmatics of collective coexistence in social worlds: acting ethically requires negotiating the complexity of human situations while honoring the experiences and the dignity of other people, recognizing the values that are being brought to bear, justifying the prioritization of values in some way that supports the well-being of the collective, taking action, and admitting that action is fallible. For the purpose of this paper, then, I submit that ethical actions are those that are considered, communicable, and conscious of the collective good.

In other words, the question of ethics is the question of how to act, in any given moment, in a way that does not simply replace one oppressive power relation with another, but rather illuminates and communicates a choice that supports the collective enterprise, even though the choice may need to be revised. This ethics calls for a daily resistance that unveils oppression and makes oppression harder to ignore, rendering it more difficult for oppression to operate, until there exists a world in which oppressive behaviors are no longer acceptable.¹⁸ It opens up the possibility of "developing new definitions of power and new patterns of relating across difference"¹⁹—of imagining a more just alternative in which every human has worth.²⁰ This, then, is a critical pragmatist ethics that is specific, contextual, and fallible, and that seeks liberation for the collective as a guiding principle, even and especially in the smallest moments.

WHAT DO I MEAN BY MATHEMATICS?

Thus far, in addition to identifying school math as one form of mathematics, we have named critical mathematics as one of many applications of school math, and also pointed to "real math." Real math, which is more creative and complex than school math, is often defined as the mathematics done by professional disciplinarians, or "real mathematicians." Mathematics education scholars, however, have traced the link between practices commonly thought of as characteristic of real mathematicians, such as abstraction, generalization, definition, a reliance on deductive logic, and claims to objective truth, and cultural practices typical of White, male, educated thinkers—the people who have, throughout Western history, been the most likely to become professional disciplinarians.²¹ Although these practices are not "bad" practices in and of themselves, we again run the risk of privileging a view of mathematics that takes for granted how one relatively small group of people does mathematics, leaving intact its formatting power and its ability to mark those who think differently as being inferior.

An important alternative perspective is ethnomathematics, which treats mathematics as a cultural practice. By doing so, ethnomathematics rejects the taken-for-granted nature of school math or even the historically-fraught tradition of "real math" and offers the existence of multiple mathematics-es.²² The mathematics of Brazilian candy sellers, for example, is not the mathematics of Yoruba accountants or the mathematics of American construction workers, and none of these are school math or "real math." All of these mathematics-es are human practices that evolved and continue to evolve within particular human cultures. Seeking mathematical practices in cultures that are not our own risks framing mathematics through a Western lens; we may think we see Cartesian geometry in Navajo rug weaving, for example, and overlook that indigenous logic is grounded in very different epistemologies. Nonetheless, ethnomathematics suggests that there is no singular mathematics, not even "real math," and that no mathematics should be held as universally superior over another. From ethnomathematics, we can take that any ethical reason to teach and learn mathematics must account for the multiplicities of mathematics and for the arbitrariness with which school math has become *the* math we teach and learn in schools.

AN ETHICAL REASON TO TEACH AND LEARN MATHEMATICS

An ethical reason to teach and learn mathematics, then, must be specific and contextual, considered and communicable, and aware of its fallibility; it must account for multiple mathematics-es and for the formatting power of mathematics, and seek liberation and well-being for the collective. Let me offer one possibility, which assumes that people encounter dilemmas as they coexist, and that negotiating these dilemmas in unoppressive or anti-oppressive ways is integral to collective liberation. In these dilemmas, people have intuitions about mathematical ideas such as enumeration (how many) and measure (how much), and the teaching and learning of considered and communicable mathematics-es is key to their ability to negotiate and act upon these intuitions.

Suppose that several of us are preparing a banquet together. Our various values might lead us to attend to different features of the context and to develop different mathematical intuitions about how many chickens to roast and how much rice (how many rice?²³) to cook. Some of us may value preparing enough food to demonstrate our prosperity as banquet hosts, and others may value the taking of as little plant and animal life as is necessary to feed everyone. Some of us might prioritize how much our guests will want to eat, and others might prioritize how much they ought to eat. To decide how much food to prepare, the most experienced, credentialed, or respected chefs among us may leverage their power to insist that we follow their intuitions. Or, we could use mathematical language and ways of reasoning about numbers and measures to negotiate action as a community and in doing so, resist the possibility for those with the loudest voices or most social cachet to coerce the rest of us into compliance. When our mathematics-es are considered and communicable, we can make collective decisions based on the defensibility of the actions, format our banquet accordingly, and reevaluate if necessary, instead of simply anointing one of us as logically or deductively or powerfully "right."

Just as there are no universal rules about whether it is okay to lie, in this ethics there are no universal rules about the best way to estimate or the best unit of measure; how one acts is specific to the preparation of this meal in this space, time, place, and with these people. Measuring water to cook rice might require different techniques than measuring water to bake a pie crust, which might depend on the day's humidity, the scale of the dishes, and our familiarity with the dishes. And we would not generalize these ways of measuring to, say, building a house or forecasting the weather. Considering the mathematics-es informing our actions and being able to communicate them to other people supports us in renegotiating the histories, norms, and structures that would otherwise re-create existing power relations in the determination of how to act and why. Lest we assume that there is a Platonic "mathematics," a core set of mathematical ideas or a mathematical way of thinking, that is identical across the various cultural practices that constitute various mathematics-es, let us take a closer look at the idea of measure. Measure seems to be common to the cultural practices of many peoples, so we might suppose that it would figure in multiple mathematics-es. What is measured, however, and how, varies widely. For example, in the U.S. we measure the worth of human life, in part, by achievement on standardized tests in school, distributing financial and other resources to schools and students based on test results. We also assess the worth of human life in terms of earning potential; insurers calculate payouts for premature death in dollars based on occupation and years of life remaining. In other cultures, human life may not be measurable in such ways or at all.

Likewise, land could be measured many ways.²⁴ A piece of land could be defined based on its longitudinal and latitudinal coordinates, based on the quantity of harvest it produces or the diversity of animal life that depends on it for sustenance, based on geological features such as mountains and rivers which may move over time, based on the claims of ancestors and first residents, or any number of other measures-and then its size measured accordingly. Rather than declaring that the ways of measure that make sense to those in power are intellectually superior to other ways, or becoming paralyzed by relativistic hypotheses that they are all equally sensible at all times, the teaching and learning of mathematics-es can support us in considering how we might choose to measure in specific contexts. What values might be involved and how could we communicate them? How could transparency and accountability in the processes of making such choices format the decision as less oppressive and more just? And what might lead us to revise a measurement? Our intuitions will be diverse; making our mathematics-es specific and contextual, considered and communicable, and subject to revision can support the human collective in negotiating among our various epistemologies and values as we aspire together towards less oppressive ways of relating.

CONCLUSION

In a complex social world where people with diverse intuitions, epistemologies, and values need to co-exist and even co-operate, ethical action would be action that supports this co-existence and co-operation. In doing so, ethical action would collectively resist power relations that oppress some intuitions, epistemologies, and values, and resist the coercion of people who hold these oppressed ideas and practices into acting like those in power. Co-existence and co-operation benefit from communicating how decisions can be and are made—decisions made in specific and contextual ways, decisions made while considering collective well-being—and acknowledging that all decisions are subject to revision. Insofar as learning multiple mathematics-es enables us to act in this way when we encounter mathematical dilemmas, I conclude, then, that the teaching and learning of mathematics—or rather, of mathematics-es—has intrinsic ethical value.

Let us return, for a moment, to the question of school math. Should we stop teaching it in favor of other mathematics-es? One might think that school math would be antithetical to the ethical stance I have adopted in this paper because it values certainty (getting the "right" answer, and often a single right answer), generalizability (finding a universal rule), and decontextualization (using abstract and symbolic representations). But that is precisely why we need to teach school math: to know what we are liberating ourselves from. The violent consequences that have resulted from school math being privileged as the only mathematics in school remind us of what happens when any single mathematics becomes the only mathematics that is taught and learned. The example of school math reveals the insidiousness of how oppression re-produces: ideas that serve a particular ideology—in this case the dominance of White, male, Western, elite cultural practices—are positioned both as neutral and as the only possible logical ideas, so their elevation seems natural rather than the consequence of oppressive systems, and their formatting power is used to justify injustice.

But does the instructiveness of school math as a case study in how invisible histories, norms, and structures sustain oppressive power relations call for the teaching and learning of school math, or just the teaching and learning of the *history* of school math? After all, focusing on history could avoid re-performing the violence of school math by teaching it. Teaching and learning merely the history of school math, however, is inadequate as an ethical act for two reasons. First, treating school math as a historical relic distances it from the formatting power it continues to hold over contemporary society. Instead, as long as school math continues to sort and commodify people, we must study it as a living artifact. Second, as reformers promote new manifestations of school math, it is worth examining whether these innovations are truly less oppressive than what they aim to replace, or whether they represent new forms of oppression. So, school math itself should not be eradicated, but rather, should be taught alongside other mathematics-es and in the context of its history of being privileged and thus its power *to* privilege.

Existing on this planet and forming relationships with others under the influence of complex histories, norms, and structures requires constant negotiation of how to act. Learning multiple mathematics-es and becoming conscious of and able to communicate mathematical reasoning and the values underlying mathematical decisions enables us to go beyond unconsciously re-producing ways of being that confer power to some intuitions, epistemologies, values, and people over others. It enables us to negotiate differences in less oppressive ways. Understanding the multiplicities of mathematics can help us to be more appreciative of and humble towards cultural practices and ways of knowing that are not our own,²⁵ and understanding the history of mathematics illustrates how systems can become oppressive despite their purported distance from politics and power. Thus, it is not only ethical but ethically necessary to teach and learn mathematics-es to support collective resistance against existing oppressive power structures.

¹ Questions which have been taken up by Mark Boylan, "Ethical Dimensions of Mathematics Education," *Educational Studies in Mathematics* 92, no. 3 (2016): 395–409; Brian R. Lawler, "To Rectify the Moral Turpitude of Mathematics Education," *Journal of Urban Mathematics Education* 9, no. 2 (2016): 11–28; and Nel Noddings, "Politicizing the Mathematics Classroom," in *Math Worlds: Philosophical and Social Studies of*

Mathematics and Mathematics Education, eds. Sal Restivo, Jean Paul Van Bendegem, and Roland Fischer (Albany: State University of New York Press, 1993), among others. 2 Kurt Stemhagen, "Doin' the Math: On Meaningful Mathematics-Ethics Connections," *The Montana Mathematics Enthusiast* 5, no. 1 (2008): 59–66.

3 See Eric Gutstein, "The Politics of Mathematics Education in the United States: Dominant and Counter Agendas," in *Culturally Responsive Mathematics Education*, eds. Brian Greer, Swapna Mukhopadhyay, Arthur B. Powell, and Sharon Nelson-Barber (New York: Routledge, 2009), and Danny B. Martin, "Researching Race in Mathematics Education," *Teachers College Record* 111, no. 2 (2009): 295–338, among others, for more detailed explanations of the relationship between mathematics education and the perpetuation of racial hierarchies and oppressive systems.

4 Michael Omi and Howard Winant, Racial Formation in the United States (New York: Routledge, 2014) call this idea producerism, and claim that it characterizes the prevalent contemporary neoliberal ideology.

5 e.g., Indigo Esmonde, "Ideas and Identities: Supporting Equity in Cooperative Mathematics Learning, *Review of Educational Research* 79, no. 2 (2009): 1008–1043; Victoria Hand, "Seeing Culture and Power in Mathematical Learning: Toward a Model of Equitable Instruction," *Educational Studies in Mathematics* 80, no. 1–2 (2009): 233–247; Joi Spencer, Dan Battey, Mary Q. Foote, Tonya Gau Bartell, Ann Ryu Edwards, and Anita A. Wager, "Equitable Instructional Practices in Mathematics: A Literature Review" (paper presentation, Annual Meeting of the National Council of Teachers of Mathematics Research Conference, San Antonio, TX, April 2017). 6 For a contrast between the neat certainty of "school math" and the complexity and beauty of what some scholars call "real math," see, for example, Holly Brewster, "Problem Solving as Theorizing: A New Model for School Mathematics," *Philosophy of Education* (2014): 169–177 and Paul Lockhart, *A Mathematician's Lament* (New York: Bellevue University Press, 2009).

7 See Jo Boaler and James G. Greeno, "Identity Agency and Knowing In Mathematics Worlds," in *Multiple Perspectives on Mathematics Teaching & Learning*, ed. Jo Boaler (Westport, CT: Greenwood Press, 2000).

8 See, for example, the work of postcolonial, queer, and indigenous theorists such as Judith Butler, Jacques Derrida, Frantz Fanon, Michel Foucault, Adrienne Rich, Edward Said, and Gayatri Spivak.

9 For mathematics as a White institutional space, see Danny B. Martin, "E(race)ing Race from a National Conversation on Mathematics Teaching and Learning: The National Mathematics Advisory Panel as White Institutional Space," *The Montana Mathematics Enthusiast* 5, no. 2/3 (2008): 387–398 and Dan Battey and Luis Leyva, "A Framework for Understanding Whiteness in Mathematics Education," *Journal of Urban Mathematics Education* 9, no. 2 (2016), 49–80. For mathematics as a form of Western cultural imperialism, see Alan J. Bishop, "Western Mathematics: The Secret Weapon of Cultural Imperialism," *Race & Class* 32, no. 2 (1990): 51–65.

10 For a compilation of readings on critical mathematics, see Anita A. Wager and David W. Stinson, eds., *Teaching Mathematics for Social Justice: Conversations With Educators* (Reston, VA: National Council of Teachers of Mathematics, 2013). For examples of how critical mathematics educators suggest teaching mathematics for social justice,

see Elizabeth de Freitas, "Critical Mathematics Education: Recognizing the Ethical Dimension of Problem Solving," *International Electronic Journal of Mathematics Education* 3, no. 2 (2008): 79–95 and Eric Gutstein and Bob Peterson, eds., *Rethinking Mathematics: Teaching Social Justice by the Numbers* (Milwaukee: Rethinking Schools, 2005), among others.

11 Audre Lorde, of course, critiques this approach as unable to bring about genuine and transformative change even if it can temporarily reverse specific power relations. 12 Renuka Vithal and Ole Skovsmose, "The End of Innocence: A Critique of 'Ethnomathematics'," *Educational Studies in Mathematics* 34, no. 2 (1997): 142.

13 For a further explanation of this limitation, see Kevin Kumashiro, "Toward a Theory of Anti-Oppressive Education," *Review of Educational Research* 70, no. 1 (2000): 25–53.

14 e.g., Peter Appelbaum, "Nomadic Ethics and Regimes of Truth," For the Learning of Mathematics 34, no. 3 (2014): 17.

15 Judith Butler, *Giving an Account of Oneself* (New York: Fordham University Press, 2005), 22.

16 Carolyn Culbertson, "The Ethics of Relationality: Judith Butler and Social Critique," *Continental Philosophy Review* 46 (2013): 458.

17 Butler, Giving an Account of Oneself, 110.

18 See Sara Ahmed, *Living a Feminist Life* (Durham: Duke University Press, 2017) for her conception of "refusing displacement" as a way of being feminist.

19 Audre Lorde, *Sister Outsider: Essays and Speeches* (Berkeley: Crossing Press, 1984), 123.

20 Emilie Townes, *The Womanist Dancing Mind: Cavorting with Culture and Evil* (New York: Palgrave McMillan, 2006).

21 e.g., Brian Lawler, "To Rectify the Moral Turpitude of Mathematics Education," *Journal of Urban Mathematics Education* 9, no. 2 (2016): 11–28; Danny Martin, "E(race) ing Race from a National Conversation on Mathematics Teaching and Learning," *The Montana Mathematics Enthusiast* 5, no. 2/3 (2008): 387–398; Alexandre Pais and Paola Valero, "Researching Research: Mathematics Education in the Political," *Educational Studies in Mathematics* 80, no. 1–2 (2012): 9–24.

22 See Ubiratan d'Ambrosio, "Ethnomathematics and Its Place in the History of Mathematics," For the Learning of Mathematics 5, no. 1 (1985): 44–48, and Renuka Vithal and Ole Skovsmose, "The End of Innocence: A Critique of 'Ethnomathematics'," Educational Studies in Mathematics 34, no. 2 (1997): 131–157 for a summary of some of the foundational ideas in ethnomathematics, and Helaine Selin, Mathematics Across Cultures: The History of Non-western Mathematics (Dordrecht: Kluwer Academic Publishers, 2000) for examples of ethnomathematical studies.

23 As Diane Nelson, *Who Counts? The Mathematics of Death and Life After Genocide* (Durham: Duke University Press, 2015) discusses, linguistic and cultural practices shape how we think about what can be individuated, apart from its relationship to a collective, and therefore counted. In English we say "a chicken" but never "a rice" or "a water" (unless, perhaps, ordering a container of water in a restaurant); we may say "a grain of rice" or "a drop of water," but never "a one of chickens."

24 Helen Verran, in Selin, Mathematics Across Cultures, cites aboriginal Australian epis-

temologies in contesting the idea that land can be owned at all, therefore eliminating one of the primary reasons that Western societies measure it.

25 e.g., José Medina, The Epistemology of Resistance: Gender and Racial Oppression, Epistemic Injustice, and Resistant Imaginations (New York: Oxford University Press, 2007).