Is Democratic Mathematics Tracking Possible? Paul Ernest's Differentiated Curriculum

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Tracking is a highly controversial topic in the education literature, and many of these debates have focused specifically on mathematics tracking as a source of inequity. Because of the growth of prestigious and well-paying careers in science and technology, mathematics has become linked to economic prosperity. Vance Martin takes this idea even further, explaining that in our technological information age, those who create and control digital information wield social and economic power.[1] This brings mathematics education to the forefront as a pathway to high-tech careers, and tracking as a potential barrier. Many concerns about equity have been raised, including questions about access to quality education and fair course placement. Paul Ernest, a philosopher of both mathematics and mathematics education, argues that the only democratic option is a differentiated mathematics curriculum based on student choice and career goals. While others have written on the issue of democratic choice in education, most do not consider mathematics specifically. Ernest, on the other hand, provides detailed recommendations for the mathematics curriculum. In this essay, I examine his definition of democratic education and his curricular recommendations for nonmathematicians. I argue that, while his version of mathematics tracking seems attractive at first, it does not satisfy his democratic goals.

Tracking systems can separate students by ability or curriculum. Ability tracking groups students by common ability levels. In secondary schools, it can appear as a parallel set of courses that cover similar content, but at different levels of difficulty. Alternatively, students might take the same courses but in different years. Some students might take a course their junior year, while others take it their freshman year. Ernest is against ability tracking, as he finds the concept of "mathematical ability" to be largely a social construction, created by students' experiences and the perceptions of others.[2] On the other hand, Ernest questions the common utilitarian justification for sending all students through the traditional mathematics sequence. Common rhetoric is that students need mathematical skills for future employment and for life, but Ernest argues that this does not justify requiring them to take several upper-level, abstract mathematics courses. He claims that even the more technical fields such as information technology, accountancy, actuarial studies, and economics do not make use of the traditional mathematics most students have learned in school. The necessary skills are instead taught in professional institutions or in practice.[3] Ernest believes that utilitarian arguments do not justify compulsory mathematics nor should they drive the general mathematics curriculum. He asserts that the need to prepare future mathematicians should not determine the curriculum for all students, because very few will pursue mathematical research. Therefore,

> what is needed is differentiated mathematics curricula to accommodate different aptitudes, attainments, interests, and ambitions. Such differentiation must depend on balanced educational and social judgements rather than exclusively on mathematicians' views of what mathematics should be included in the school curriculum.[4]

Students should all share the same curriculum in elementary school, but once they have "acquired basic mathematical competency," they should be allowed to choose courses based on their interests and future plans, or stop taking mathematics all together (*WTM*, 13).

As a solution, Ernest proposes a form of curricular tracking for mathematics in secondary school. The choice would be based solely on student interests and goals, not on any measurement of student mathematical ability. There would be a track for future mathematicians, which would teach students advanced mathematical content and familiarize them with the process of conjecture, proof, and public critique through which mathematicians create new mathematical knowledge. Students would be prepared to eventually join the community of research mathematicians. Ernest envisions separate tracks for other career goals. This might include different college preparatory tracks for mathematics-related fields such as engineering or information technology, as well as nonmathematics fields such as the humanities, and vocational tracks for students not planning to go to college. Ernest outlines a core curriculum that he feels all of these students should encounter, which fosters mathematics appreciation and critical mathematical awareness through an investigatory pedagogy.

Can curricular tracking based on student choice provide an equitable, democratic mathematics education? Ernest feels that it does not suffer the same downfalls as ability tracking. However, both types of tracking have been heavily criticized in the literature as inequitable. Many studies conclude that low income students and racial minorities are disproportionately placed in lower tracks, where they often receive an inferior mathematics education.[5] Jeannie Oakes cites her own research as well as many other studies which indicate that lower-track mathematics classes cover less demanding topics with less emphasis on problem-solving tasks, focusing instead on fragmented chunks of material and rote memorization.[6] Findings such as these have led many researchers to believe that the disadvantages to students in the lower tracks are not worth any advantages that might be reaped by students in the higher tracks.

There is no easy solution, however. Requiring all students to take a college preparatory sequence of advanced mathematics courses is also unattractive. Students who are not training to be mathematicians are then required to take courses well beyond the level required for their goals, possibly at the expense of more relevant courses. Ernest is trying to avoid this, because he sees it as undemocratic. Advanced mathematics courses would not provide much benefit, in terms of utility or even personal enrichment, to students going into careers that are not mathematics intensive. Even courses earlier in the sequence would include more abstract topics, language, and procedures. This will bore some of the students who do not want to be mathematicians, and some will struggle with these more formal features of the curriculum. In both cases, these students could become frustrated and therefore miss the larger concepts. It is possible they would benefit more from a course that takes their interests into consideration, focusing on the larger concepts without addressing unnecessary procedural details. Ernest attempts to remedy this problem by providing curricular choice, while also ensuring that all students, in all tracks, receive a high quality mathematics education. Next, I outline his recommendations, and then analyze them for consistency with his democratic principles.

Ernest's Differentiated Curriculum

Ernest's views on mathematics curricula are rooted in a philosophy of mathematics education that combines a form of social constructivism with a particular set of democratic values. My intention is not to evaluate this vision of democratic education, but to examine his differentiated curriculum for consistency with it.

Ernest's philosophy incorporates what he calls the "ideology of public educators," which provides a set of values and aims. As Ernest describes it, the public educator ideology in mathematics education is based on the general aims of democracy and social equity, to be achieved through the development of critical thinking. The goal is not just to create citizens who can participate in a democratic society, but to empower learners and encourage autonomy.[7] This goes beyond a utilitarian view of economically productive citizens who function efficiently within the parameters of their social status. To describe Ernest's conception of democratic citizenship I borrow from Eric Gutstein, who differentiates between two kinds of mathematical literacy: functional and critical. Functional literacy prepares people to live and work in modern society, but perpetuates social reproduction. Critical literacy includes mathematical content, but also prepares citizens to examine sociopolitical contexts and to recognize "oppressive aspects of society."[8] The image of a democratic citizen that underlies Ernest's general curriculum is someone with critical literacy — someone who understands the importance of mathematics in many areas of life and society, and is empowered by this knowledge. Ernest seeks three forms of empowerment: (1) mathematical empowerment, or mathematical ability; (2) social empowerment, or the ability to use mathematics in one's life and to critique its use in the world; and (3) epistemological empowerment, or confidence in one's abilities and a personal sense of power.[9] Together, Ernest believes that these forms of empowerment help to create a critical citizen who is able to interpret and critique the mathematics inherent in political and social systems and used in claims in the media, and who, when necessary, will act toward social change.

Ernest bases his curriculum on the general democratic principles in his philosophy of education: equity, diversity, social empowerment, and social justice. In this context, Ernest's conception of the democratic ideal is focused on entitlement and empowerment as markers of equity and social justice. Ernest believes that in a democratic society, students in secondary school should be given a choice about whether to continue their mathematics education, and in which direction. "If education is to contribute to the development of autonomous and mature citizens, able to fully participate in modern society, then it should allow elements of choice and self-determination" (*WTM*, 13). He asserts that students are entitled to a mathematics education that provides them only with the material they need, but is also meaningful, presenting them with important mathematical ideas and helping them become empowered and critical democratic citizens. Ernest recommends that the mathematics curriculum include specialized tracks for different interests and career goals, with a common core centered on the themes of mathematics appreciation and critical awareness.

Mathematics appreciation, by Ernest's definition, requires that students be able to think mathematically. This would require an understanding of the main branches of mathematics and their connections, with an emphasis on "big ideas," or interesting concepts in mathematics that affect our universe, such as infinity, symmetry, chaos, and randomness (*WTM*, 9). Ernest acknowledges that students will need some level of mathematical competency to work with these concepts, but he does not think that this competency will need to be very advanced. For example, a person can comprehend the idea of infinity without knowing calculus, or randomness without being able to calculate probabilities. Some mathematical capabilities will be needed, however, and they should be presented when necessary.

Another important part of mathematics appreciation, in Ernest's opinion, is an awareness of the historical development of mathematics and the different philosophical views about its nature (*WTM*, 9). Students should learn about the social contexts in which mathematics has developed and the debates over its foundations. In addition, mathematics should be presented as an integral part of human culture "with its own aesthetics and beauty" (*WTM*, 13). Ernest hopes that students will appreciate its role in philosophy, art, science, technology, and other areas. This will serve to humanize mathematics, connecting it to society and social issues.

The final aspect of Ernest's mathematics appreciation is an awareness of mathematical thinking in everyday work and life, as well as the mathematics that permeates social and political systems. Ernest wants students to recognize the part mathematics plays in their lives, and the mathematical thinking that is implicitly required of them in the course of their daily activities. In other words, students should understand that mathematics is all around them, and should be able "to identify, interpret, evaluate and critique the mathematics embedded in social and political systems and claims, from advertisements to government and interest-group pronouncements" (*WTM*, 9). Possible topics could include the mathematics behind voting systems or judging the validity of statistical interpretations. This part of the curriculum is meant to involve both critical awareness and social empowerment.

Ernest also recommends a particular pedagogy, which combines his democratic aims with a social constructivist learning theory. His social constructivism is based on the idea that language is the medium for learning and concept formation, and thus for the genesis of higher level thought. Therefore, the learning process involves conversation and interaction with others. The construction of a concept is a social rather than individual process, in which interaction allows the individual to negotiate a fit between her understanding and the knowledge of others. Students need experiences that allow them to construct and refine mathematical concepts by discovering connections and testing ideas in new contexts. Conversation and interaction also allow students to compare their ideas and test their validity. Through dialogue, students "generate, test, correct, and validate mathematical performances" to ensure that their constructions are consistent with those of others and with the mathematical knowledge base.[10] Thus, students need to be actively engaged in discussions about mathematics, challenging one another's ideas and confronting different perspectives.

To provide opportunities for the social construction of mathematical concepts within a democratic context, Ernest recommends an investigatory approach to teaching mathematics that models the way mathematicians create new mathematical knowledge. He explains that investigations are similar to problem solving approaches, but are more open ended. Ernest sees problem solving as an activity in which the teacher poses a problem, and then leaves the students free to find their own methods for solving it. In an investigatory approach, the teacher merely provides a beginning situation for students to consider. This is only the starting point for the investigation — if students are to be truly engaged and encouraged to think critically, then they must be allowed some latitude to explore different aspects of the topics presented. It is up to them to define a problem on which they would like to focus. Ernest sees investigations as essentially similar to problem solving, with the crucial addition of a problem-posing step.[11] Therefore, he often refers to the investigatory approach as "problem posing and solving." He feels students should be encouraged to pose problems about mathematical concepts as well as mathematical contexts in society. This will help them to learn both the mathematical content and the critical awareness that Ernest values. For Ernest, problem posing and solving are at the heart of mathematical learning, incorporating conversation, mathematical discovery, and mathematical and democratic empowerment.

On the surface, at least, Ernest's vision alleviates some of the concerns about tracking. Because students would choose their track based on their interests, they would not be held back by perceptions about their ability. Content would be tailored to student goals, without offering a less demanding curriculum that covers less mathematical content. All courses would provide meaningful, thought-provoking content relevant to the students' lives or future careers. All students, regardless of curricular track, would engage in independent thought and problem solving. Ernest feels that his curriculum would provide a more democratic method of course selection, eliminate the overrepresentation of low income and minority students in vocational tracks, and remedy the inferior pedagogy based on rote memorization and meaningless procedure that students in vocational tracks often encounter. However, he undermines his own democratic goals with mathematical elitism, and he ignores the impact power structures have on student choice.

Mathematical Elitism

Ernest's tracking recommendations seem to value the legitimacy of different educational and career options, allowing all students to receive a meaningful, relevant mathematics education. However, a closer look at his differentiation between future mathematicians and everyone else reveals an underlying conception of mathematical knowledge that excludes nonmathematicians.

For Ernest, mathematical knowledge is a commodity that empowers the one who possesses it. This happens on two levels. On a personal level, mathematical knowledge gives students a sense of accomplishment, and provides access to higher education and better employment opportunities. On a social level, mathematical knowledge provides a conceptual background that allows students to analyze and critique the social uses and implications of mathematics. It enables them to recognize when mathematics is being used, either intentionally or not, to discriminate or to manipulate the public. This recognition could be an alertness to the social implications of the mathematics embedded in a voting system, or an awareness of statistics being purposefully manipulated in the media to discriminate or to make a political point. Ernest envisions mathematical knowledge as a force that empowers students to call attention to such issues and to fight for social justice.

The actual content of mathematical knowledge, however, privileges research mathematics. One of Ernest's stated intentions is to show students how mathematics is intrinsically interesting and permeates the world around us, but this uncovers a rather traditional aspect in his view of mathematical knowledge. While he wants students to choose their courses based on interests and career goals, he believes the curriculum should include topics from pure mathematics. The mathematical capability necessary for research mathematics may be de-emphasized, but he presents mathematics as the overarching force that controls every aspect of the world and our lives. Thus Ernest's conception of mathematical knowledge holds on to a view of mathematics as a privileged discipline, and it conveys the impression that mathematics is the sole driving force in the universe.

Ernest does want students to recognize mathematics as part of culture, to connect to mathematics in a meaningful way and to develop an awareness of its role in our daily lives. He believes that students should have a sense of how mathematics "permeates and underpins" human culture, be able to identify the mathematics "embedded in social and political systems and claims," and even develop general problem solving abilities (*WTM*, 9–10). But mathematical knowledge does not seem to include practical applications for life or work. Procedures such as balancing a checkbook or designing a stable bridge are peripheral to a theoretical understanding of the mathematical principles involved. The emphasis is on recognizing the existence of applications and interpreting their results, but the applications themselves are seen as uses of mathematics rather than mathematical knowledge in their own right.

Ernest also reveals an underlying conception of knowledge through his investigatory pedagogy. In the mathematical community, Ernest associates problem posing and solving with the creation of mathematical knowledge. This is a process in which one investigates a question, proposes a potential solution, tests and revises it in response to counterexamples, publishes a formal exposition of the final approach, and uses public feedback to begin the process again. Problem posing and solving is the mathematical inquiry leading up to formal proof and publication, providing the creative mechanism in this cycle. Ernest does not include in the general curriculum any practice with the final step, writing up formal proofs. He sees these students as engaging in mathematical activity, but not formal justification. This indicates that he does not view these students as future producers of new mathematical knowledge. His goal is to create informed consumers of mathematics who have some sense how the mathematical community operates, but are kept distanced from its activities. Mathematics is not an end in itself for these students, but familiarity with it provides a means to social and intellectual empowerment.

Ernest's underlying assumption is that a mathematician is one who has completed a doctoral program and participates in research mathematics. He does not see others who use mathematics in their careers, such as engineers, economists, or even secondary school mathematics teachers, as part of the mathematical community. Their mathematical activities are not considered part of "real" mathematical practice. Research mathematicians are seen as the only creators of mathematical knowledge, and the production of mathematics is valued over its use. Ernest does feel that mathematical knowledge has intrinsic value for all citizens. Mathematics is embedded in many aspects of our lives and culture, sometimes with negative implications. The role of the democratic citizen is to be aware of these implications and to work for social change. However, Ernest calls for a system of mathematics tracking that separates future mathematicians from other students, providing a general mathematics curriculum that does not engage students in what he considers real mathematical work. He encourages nonmathematicians to become personally and socially empowered, but excludes them from the mathematical community. His visions of equity and social justice are undermined by the power differential created by the insider–outsider roles inherent in his curricula.

Power Structures

In addition to the power differential he inadvertently creates between mathematicians and others, Ernest does not consider the impact existing power structures have on students and their educational choices. One concern is Ernest's use of language and conversation as the medium for learning. Ernest proposes to achieve democratic aims through the use of his investigatory pedagogy. Conversation serves as the basis for these investigations, providing the means for students to carry out mathematical inquiry, become more confident in their own skills, and uncover social inequity in situations that often go unquestioned. Therefore, conversation plays an emancipatory role, fostering democratic citizenship by developing both mathematical and social empowerment in students. However, conversation can be influenced by the very social norms that have caused students to overlook social issues in the past. As Jürgen Habermas warns, "Language is *also* a medium of domination and social power. It serves to legitimate relationships of organized force."[12] Ernest recommends that teachers make these issues explicit in the classroom, encouraging conversations about cultural domination and related social problems. However, Ernest needs to rely on more than exposure to social issues. He must offer suggestions as to how students can be encouraged to reflect critically on their own presuppositions and assumptions in the course of classroom conversations.

The impact of power structures begins before students even enter their mathematics classroom, however. Ernest envisions a curricular tracking system in which student interests prevail, but this seems unrealistic. Student choices can be unduly influenced by the recommendations of teachers and counselors throughout a student's educational career. In a review of research, Oakes found that many non-Asian minority students leave the "scientific pipeline" in elementary school. Even this early, they are more likely to be placed into special education and less likely to be placed into enriched programs.[13] Minority students are also more likely to attend poor schools that suffer from a lack of school funding, which results in inferior opportunities and educational quality.[14] The elementary school experience of many low income and minority students restricts the mathematics courses they are prepared to take in middle and secondary school. Even without the issues of secondary tracking, these students are already less likely to be successful in their mathematics classes.

On reaching secondary school, counselors are unlikely to encourage students to make up these deficiencies. In a case study of three high schools in California, Jeannie Oakes and Gretchen Guiton found that many teachers and counselors thought of Latinos, and to a lesser extent African-Americans, as better served by occupational training than academic coursework because these students were not likely to go to college.[15] Other studies have found that low income and minority students receive limited academic advice about course options, and that women and minorities receive less encouragement to pursue careers in scientific fields.[16] These perceptions can be quite influential in how young people see themselves, undermining the possibility of true democratic choice. Rather than making decisions based on their preferences, fully informed about educational and career options, students are nudged toward a track based on stereotypes. These students are much less likely to choose a mathematics-intensive course of study, and worse, Ernest would allow them to opt out of mathematics all together. It seems like social justice would be better served by encouraging students to continue in a mathematics track that would keep their future options open. Robert Moses' Algebra Project, for example, was built on the premise that algebra is the great gatekeeper course to college, that minority students have been steered away from it due to low expectations on the part of students and educators alike, and that these perceptions can and must be changed.[17] While Ernest would argue against the relevance of algebra to many career goals, this illustrates that there are other realities that must be considered. In addition, allowing students to opt out of mathematics in Ernest's system would deprive them of the critical awareness and social empowerment aspects of his curriculum, as well as the opportunity to discover interests that have been suppressed by earlier experiences. In this case, Ernest's principle of democratic choice is inconsistent with his desire to overcome social inequities in mathematics education. Rather than perpetuate the cycle, it seems that Ernest should want the secondary curriculum to encourage women and minorities to explore avenues that have previously seemed closed.

As far as tracking options go, it is possible that Ernest's design would avoid some of the pitfalls of traditional curricular tracking. He has attempted to take the needs of all students into consideration, and has designed a curriculum intended to empower those who are usually disadvantaged. However, he has not provided any strategies for dealing with the existing power structures that restrict mathematical choices in the first place.

Conclusion

Ernest attempts to outline a differentiated mathematics curriculum that allows students to have democratic choice, and empowers them mathematically and socially. He hopes to overcome any equity concerns by designing a curriculum that provides all students with quality mathematical content. There are promising aspects to his design, but his curricular tracking sets up a power differential between mathematics insiders and outsiders that is not consistent with his democratic goals. It also allows students who are already socially disadvantaged to end their mathematics education after elementary school, rather than attempting to counteract the negative effects of previous social and educational experiences. In the end, these issues undermine his vision of providing democratic mathematics education by offering choice and fostering critical literacy. It seems that any tracking proposal would create inherent differentiations and rankings between students, making it impossible to reconcile tracking with democratic education as Ernest envisions it.

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2. Paul Ernest, *The Philosophy of Mathematics Education* (London: Falmer Press, 1991), 208.

3. Paul Ernest, "Why Teach Mathematics?" in *Why Learn Maths?* ed. Steve Bramall and John White (London: University of London Institute of Education, 2000), 3. This work will be cited as *WTM* in the text for all subsequent references.

4. Paul Ernest, *Social Constructivism as a Philosophy of Mathematics* (Albany: State University of New York Press, 1998), 273.

5. Jeannie Oakes, *Multiplying Inequalities: The Effects of Race, Social Class, and Tracking on Opportunities to Learn* (Santa Monica, Calif.: RAND Corporation, 1990), 18.

6. Ibid., 81 and 89.

7. Ernest, *Philosophy of Mathematics Education*, 206.

8. Eric Gutstein, *Reading and Writing the World with Mathematics: Toward a Pedagogy for Social Justice* (New York: Routledge, 2006), 5–6.

9. Paul Ernest, "Empowerment in Mathematics Education," *Philosophy of Mathematics Education Journal*, no. 15 (2002), under "What Is Empowerment?" http://www.people.ex.ac.uk/PErnest/pome15/empowerment.htm.

10. Ernest, *Social Constructivism*, 221.

11. Ernest, *Philosophy of Mathematics Education*, 286.

12. Jürgen Habermas, *On the Logic of the Social Sciences*, trans. Shierry Weber Nicholsen and Jerry A. Stark (Cambridge, Mass.: MIT Press, 1988), 172 (emphasis in original).

13. Jeannie Oakes, "Opportunities, Achievement, and Choice: Women and Minority Students in Science and Mathematics," *Review of Research in Education* 16 (1990): 160.

14. Ibid., 180.

15. Jeannie Oakes and Gretchen Guiton, "Matchmaking: The Dynamics of High School Tracking Decisions," *American Educational Research Journal* 32, no. 1 (1995): 17–18.

16. Oakes, "Opportunities, Achievement, and Choice," 184.

17. Robert P. Moses and Charles E. Cobb, Jr., *Radical Equations: Civil Rights from Mississippi to the Algebra Project* (Boston, Mass.: Beacon Press, 2001).

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