Some Initial Steps Toward a Much-needed Critical Epistemological Realism in Mathematics and Science Education

Dennis Lomas
Ontario Institute for Studies in Education of the University of Toronto

Introduction

In the past couple of decades or so, brands of extreme relativism in philosophical discourse pertaining to mathematics and science education have arisen, some of which have considerable influence.[1] In my usage, the term "extreme relativism" is intended to refer to currents of thought which hold that *no* proposition from mathematics and science is objective, that is, (by my usage of the term "objective") no proposition is true independent of our thinking — either individually or collectively — that it is true. These extreme relativist views, furthermore, tend to be unabashed. There is no mistaking their rejection of the idea that mathematical and scientific knowledge is objectively based.[2] This relativism has some roots in mathematics, science, and philosophy, which this essay touches on. An unhappy consequence of this extreme relativism is raised: the direct experience of many students seems to be ignored.

A realism is developed which provides an alternative to this relativism. It is contended that a simple pragmatism and, crucially, the general correctness of perception provide plausible reasons for rejecting this extreme relativism, and for situating school mathematics and science within the compass of objective truth. I focus somewhat on mathematics education, but my discussion includes science education, and my conclusions apply to both. Although I reply to extreme relativism in the philosophy of mathematics and science education, the deeper purpose of this essay is to address complicated issues concerning how beliefs regarding propositions in school mathematics and science are justified, including how these beliefs are justified by mathematics and science students.

Relativism in Mathematics and Science Education

It could be argued that questioning the objective truth of at least some mathematics and science has some roots in doctrines within these fields. A couple of examples illustrate this point. The entities in modern set theory in mathematics may not be real things, but rather mere fictions which are born of, and sustained by, acceptance in the practice of mathematics.[3] While this acceptance may arise from rational discourse within mathematics practice, it is not clear that this is enough to establish objectivity. In physics, the Heisenberg uncertainty principle prompts some to suspect all knowledge derived from empirical observation.

Trends in philosophy have fed the extreme relativism in philosophy of mathematics and science education. In the last few decades a pall has fallen over attempts to find ways of showing that beliefs are objectively justified. Attempts to do so are often thought of as incoherent or irrelevant, or described in other disparaging ways.[4] Accounts of perceptual justification of belief have been widely challenged (as I point out more extensively below). In philosophy of science, the doctrine of theory ladenness of observation has wielded considerable influence. By its lights, what an observer perceives is essentially inferred from whatever theory an observer (for example, a scientist) believes.[5] Perception cannot justify belief because perception is inferred from belief. These trends aimed at justificatory strategies have helped, it would seem, clear the ground for the emergence of extreme relativism in philosophy of mathematics and science education especially because going this route seems to merge with broadly emancipatory aims.[6] For example, an extreme relativism, some might argue, is a well-suited philosophical base from which to critique the institutions and practices of mathematics and science, in general and with regard to education.

Whatever positive merit extreme relativism might have, I maintain that it has a big failing. It does not adequately capture the experience of many students. Granted, extreme relativism might seem to be appropriate for capturing the experiences of students who regard school mathematics and science as enterprises that are arbitrarily foisted on them by oppressive authority. (They probably have good reasons for thinking so.) Some students, however, bring their own mental resources to bear on justifying the mathematics and science they learn. In fact, in mathematics and science education in general, students are often provided with the means to justify the propositions which they learn. In school geometry, students learn proof techniques, and school science experiments are conducted which verify scientific theories. The extreme relativism at issue conflicts with the convictions of these students with regard to the propositions they have themselves verified, sometimes with "their own eyes" in classroom experiments.

A potential problem with extreme relativism pertains to teaching. Although in general we should be wary of drawing pedagogical conclusions from philosophical arguments and positions, it would seem that teaching mathematics and science would become problematic if teachers in these fields endorsed extreme relativism. The question would be raised: what would motivate them to teach the material? (In any case, many, perhaps most, mathematics and science teachers endorse the subject matter as objectively true.)

An extreme relativism, even if engaged in worthwhile critiques, can backfire despite the best of intentions because the relativism goes too deep. A problem of this type is discussed by the feminist philosopher Sally Haslanger in relation to social constructivism.[7] This view performs the valuable service of demonstrating that concepts such as gender are not eternal, god-or-goddess-given truths, but fundamentally derive from social institutions and practice. However, she argues that an extreme social constructivism in which all concepts are thought to derive solely from social factors (social constructivism "all the way down") becomes problematic. Physical abuse to women, she observes, would still be just that even if societal influences led to calling it something completely different. Not calling it physical abuse would basically deny the all-too-real, experienced abuse.[8] Of course, the situation of students in mathematics and science classes is completely different from that of battered women. Nonetheless, there is a parallel insofar as the problems of extreme relativism are concerned. Extreme relativism may wrongly stand opposed to the strong convictions of many students (and teachers) based on their own experience with verifying mathematical and scientific propositions. A way out of this dilemma, now pursued, is to provide a justification for beliefs pertaining to the subject matter of school mathematics and science. (This justification, if it can be shown, would help establish a realism with regard to school mathematics and science.) Narrowing the domain to *school* mathematics and science seems to lessen what needs to be shown. In particular, it at least opens up the possibility that the really difficult justificatory issues can be set aside. Students are not being asked to learn set theory or quantum mechanics. School mathematics is geared to entry level requirements of a broad range of occupations and of programs of study in higher education. Except in rare cases, it is really an elementary mathematics tied to elementary science, engineering, and finance, and only distantly related to the mathematics of set theory.

It is far beyond the scope of this essay to produce a *complete* theory of objective justification of belief pertaining to school mathematics and science. Only a sketch of a partial theory is possible. I have chosen to concentrate on two aspects of justification, both of which I explain below. The first I call simple pragmatism and the second I call perceptual justification of belief. Although other justificatory approaches deserve attention, these two are chosen because, in my view, they are key to developing a justificatory approach to mathematics and science education that is in touch with the experience of students and with how students justify the propositions which they learn.

A simple pragmatism — if it works, it is true — overwhelmingly suggests (at least to a first approximation) the objective correctness of school mathematics and science.[9] Take, for example, the case of the Pythagorean theorem. Throughout history and up to the present, it has been used extensively in scientific and engineering work. It is used today in calculations for bridge designs, in rocketing to Mars, and so on. The success of the engineering and scientific projects for which the Pythagorean theorem is indispensable provides convincing empirical evidence of the truth of the theorem.

The warrant given the Pythagorean theorem and enumerable other propositions from school mathematics and science by successful use made of them serves, it would seem, to put extreme relativism regarding the propositions from these areas on the defensive. This warrant also serves to put us in the mind set of students and teachers for whom the propositions from school mathematics and science are objectively justified by the successful use made of them. (Quantum mechanics and relativity theory which supercede Newtonian physics, generally the physics which pre- and non-university students are taught, does not undermine the simple pragmatism at issue. Newtonian physics is still a good approximation.) Thus, a simple pragmatism can be at the heart of strategies which attempt to justify the objective character of propositions from school mathematics and science.

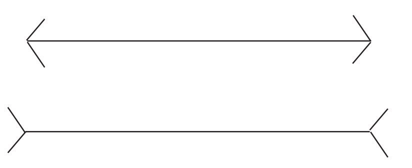
Another way of justifying school mathematics and science is to show that perceptions justify many beliefs, specifically for the present context, beliefs regarding the results of empirical testing which provide objective evidence for the propositions of school mathematics and science.[10] In fact, *perceptual* justification of belief is crucial if a realism is to be established. If such justification is denied, it is extremely hard to argue for any robust objective justification of belief. It is even hard to make simple pragmatism stick. Furthermore (and germane to my present concern), like simple pragmatism, perceptual justification directly pertains to how students assess whether the propositions from school mathematics and science are objectively true. With this in mind, the key question is: Does perception accurately deliver information to the subject to justify beliefs based on this information, such as where an indicator points in a classroom experiment? Put more loosely, can we "trust our eyes" to sustain our beliefs? It might seem that in general we obviously can, but in philosophy providing an account of the perceptual justification of belief faces many obstacles, as already indicated.

My account of perceptual justification begins with the notion of causal reliabilism which locates the source of perceptual justification of belief in the causal mechanisms which reliably capture and preserve distal information and use it to induce perception.[11] (I have in mind the case of visual perception.) So, when we see a circle, causal processes involving light reflection, and so on, capture, preserve, and deliver correct information to the retina, and other causal processes then delivery the information to the visual cortex where the inducing of *correct* perception of the circle can take place. Perception is thus able to supply correct information about our surroundings. In particular, perception is able to supply correct information that can be used to justify propositions in school mathematics and science. Consider an empirical way to justify the Pythagorean theorem. A student can use visual perception to check that a triangle with sides of 3, 4, and 5 units contains a right angle, as it should according to the theorem. Perception of experimental data — colour of chemicals, indicator lights, dials, printouts — provides evidence for the truth of scientific propositions that are tested in classroom experiments. These examples indicate the power of the appeal of causal reliabilism.

The ultimate philosophical success of causal reliabilism depends on whether it can withstand criticisms levelled against justificatory accounts that appeal to causal processes within perception. I have responded to the main criticisms elsewhere.[12] Here, I sketch only some of these criticisms and my responses to them.

One type of criticism stems from Wilfrid Sellars's influential critique of empiricism.[13] By Sellars's lights, the empiricist thought of situating the ground for a subject's beliefs in the senses is mistaken, even incoherent, at its core. Empiricism holds that causally induced, (non-inferential) sensations (so-called sense data) provide justification for belief because they are self authenticating and indubitable. The nub of Sellars's response to empiricism seems to be this. These sensations cannot justify a subject's beliefs because the sensations are *merely* causal; that is to say, the subject's knowledge base, on which classification must be based, has no bearing on the sensations. The sensations are, in a word, *non-conceptual*. Belief, however, requires *conceptual* justification.[14] Causal sensations, then, are not the type of thing that can justify beliefs. If the perceptions, to which causal reliabilism adverts, were *merely* causal they would seem to be susceptible to this Sellarsian critique. (A perception-sensation distinction is not an issue here. Merely causal perception could be called "merely causal sensation" without changing anything of substance in my discussion.) To respond to this argument, while still adhering to the main principle of causal reliabilism, the following move can be made. One can argue that the causal processes in the brain which preserve distal information and induce perception also have access to a perceiving subject's encoded knowledge base in the neuronal circuitry in the brain. Consequently, these processes are not *merely* causal. So the perceptions at issue stand in contrast to the non-conceptual sensations of empiricism. There is now sympathy in current opinion in vision science for this way of thinking about visual perception.[15] In brief, the Sellarsian challenge to causal reliabilism can be met by showing that the information-perserving processes (responsible for inducing perception) are also engaged with the perceiving subject's encoded knowledge base. The induced perception is not only reliable but also conceptual. Because perception is conceptual, it is able to provide justification for beliefs in a way that avoids the Sellarsian worry.

Another battery of criticisms against the notion of perception justification of belief stems from the doctrine of theory ladenness of observation, already mentioned. Perception, it is alleged, is never independent of belief because it is inferentially derived from a system of beliefs (a theory). The theory determines the-perception. If this were true, perceptions could not be belief-independent warrants for beliefs because perceptions would be derived from beliefs. The (justified) cart would be put before the (justifying) horse. (This doctrine of theory ladenness of observation is relevant to justificatory issues pertaining to school mathematics and science. It could be charged in line with this doctrine that, for example, in experiments students are able to see only what should be seen according to the theory which they have been taught.) The doctrine of theory ladenness of observation essentially relies on perceptual inference theory which avers perceptions are *inferred* from background beliefs and retinal stimulations. A traditional response to perceptual inference theory adverts to the persistence of perceptual illusions despite beliefs to the contrary.[16] We, for example, experience the horizon lines of the Müller-Lyer diagram (below) as unequal in length despite a firm belief (derived from measuring the lines) that they are equal in length.



If perception were inferred from belief, we would perceive the lines as equal. This observation makes a case against the doctrine of theory ladenness of observation by undermining its key premise that perception is due to inference.[17]

A third argument against accounting for objective perceptual justification of belief denies that perception provides certainty. It could be argued that causal preservation of information is not enough to guarantee veridical perception. The preserved information might induce illusion, as when we look into a mirror. Wall mirrors make restaurants seem larger than they are. At the extreme, it is not difficult to conceive of a world of "smoke and mirrors" in which all perceptions are illusory. So-called virtual reality devices may not be far from producing such an effect. A justificatory account, however, need not depend on, or subscribe to, the claim of perceptual certainty. It is enough to demonstrate that perception is sufficiently correct, due to causal preservation of information, to provide a broad basis for objectively justifying belief. Were it not true that perception is sufficiently correct in this regard, how humans in their evolution managed to avoid predators and find sustenance would surely be inexplicable.

The fact of perceptual justification of belief (if, as I have argued, it is a fact) highlights the bind extreme relativism can get into by denying the objective truth of all propositions from mathematics and science. This current of philosophical thought could well be denying what many students really do correctly see with their own eyes when they perceive the outcomes of experiments.

In responding to extreme relativism and in beginning to develop a realism regarding school mathematics and science, I have emphasized one aspect of the classroom situation. I have not focussed on phenomena such as acquiesce in the face of mathematical and scientific authority, uncritical acceptance of propositions, undue emphasis on examination performance at the expense of criticality, and a host of other issues. All of these concerns need to be taken up in the development of a rounded philosophical stance applicable to mathematics and science education.

As another clarification, I should emphasize that just because school mathematics and science, at least much of it, receives warrants both from simple pragmatism and, crucially, from reliable perception, this in no way serves to warrant *all* mathematics and science — specifically those propositions not accessible to these types of justification, of which there are many. I suspect many school mathematics and science students and teachers uncritically accept some propositions from mathematics and science. The proper response to such attitudes should not involve, as in the case of extreme relativism, rejection of all propositions of mathematics and science as objective truths, but should involve a measured criticality, which is prepared to question propositions from mathematics and science, and which has a broad range of justificatory resources at its disposal.

As a final clarification, although I argue that perception and simple pragmatism provide justification for school mathematics and science, it is not my view that they provide *full* justification, even in this domain. Other ways in which beliefs are justified need to be included in a full account of the justification of belief. These ways would include historically derived practices and activities in science and mathematics, such as refereeing, which sanction results in these fields. Nonetheless, showing that perception and simple pragmatism objectively justify propositions from school mathematics and science is a key starting point.

1. This assessment is shared by a number of papers presented in the Philosophy of Education Society in recent years. See, for example, Michael R. Matthews, "Social Constructivism and Mathematics Education: Some Comments," in *Philosophy of Education 1999*, ed. Randall Curran (Urbana, Ill.: Philosophy of Education Society, 2000), 330-41, and Christine McCarthy and Evelyn Sears, "Science Education: Constructing a True View of the Real World," in *Philosophy of Education 2000*, ed. Lynda Stone (Urbana, Ill.: Philosophy of Education Society, 2001), 369-77. The above authors refer to various brands of constructivism. For this paper, I avoid the designation "constructivism" even though some constructivists are extreme relativists, because the term refers to many positions, some quite different from others, and some more, and others less, relativist. Social constructivism, often considered a brand of constructivism, is mentioned briefly later.

2. For example, Ernst von Glasersfeld remarks in "Introduction: Aspects of Constructivism," in *Constructivism: Theory, Perspectives, and Practice*, ed. Catherine Twomey Fosnot (New York: Teachers College Press, Columbia University, 1996), 3: "[W]hat we call knowledge does not and cannot have the purpose of producing representations of an independent reality, but instead has an adaptive function." Paul Ernest in his *Social Constructivism as a Philosophy of Mathematics* (New York: SUNY Press, 1998), 276, writes: "The mathematician is omnipotent in the virtual reality of mathematics, although subject to the laws of the discipline." This is to say (it would seem) that mathematical knowledge is strictly relative to the will and power of mathematicians who are subject only to "the laws of the discipline." In another work, *The Philosophy of Mathematics Education* (London: Falmer Press, 1991), 45, Ernest remarks: "[W]e shall adopt a necessary condition for objectivity, social acceptance, to be its sufficient condition as well."

3. Philip Kitcher, "Mathematical Naturalism," in *History and Philosophy of Modern Mathematics*, ed. William Asprey and Philip Kitcher (Minneapolis: University of Minnesota Press, 1988), 315, writes: "[T]here is no independent notion of mathematical truth and mathematical progress that stands apart from the rational conduct of inquiry and our pursuit of nonmathematical ends, both epistemic and nonepistemic."

4. For example, John R. Searle in "Animal Minds," *Midwest Studies in Philosophy* 19 (1994): 217-18, contends: "Descartes together with the British empiricists and right up through the Positivists and the Behaviorists of the twentieth century have given us the impression that the question: 'How do you know?' asks the fundamental question, the answer to which will explain the relation between us as conscious beings and the world. The idea is that somehow or other we are constantly in some epistemic stance toward the world whereby we are making inferences from evidence of various kinds.… Against this tradition, I want to say that epistemology is of relatively little interest in philosophy and daily life. It has its own little corner of interest where we are concentrating on such things such as how to understand certain traditional skeptical arguments, but our basic relationships to reality are seldom matters of epistemology."

5. W. Martin Davies in *Experience and Content: Consequences of a Continuum Theory* (Aldershot: Avebury, 1996), 194, aptly describes the notion of theory ladenness of observation as "the move to *conflate* observational and theoretical terms, and to make the origin of their meaning come from the one and the same theoretical (and hence, *inferential*) source."

6. For my purposes, not much hangs on my term "emancipationist." Other terms such as "liberal" or "critical" would probably do as well.

7. Sally Haslanger, "Object Reality, Male Reality, and Social Constructivism," in *Women, Knowledge, and Reality: Explorations in Feminist Philosophy*, 2d ed., ed. Ann Garry and Marilyn Pearsall (New York: Routledge, 1996), 84-107.

8. Ibid., 84-85, observes: "Dramatic claims rejecting the legitimacy of such notions as 'truth' and 'reality' do appear in the work of feminist theorists, yet one also finds there a deep resistence to slipping into any form of idealism or relativism."

9. Because the main concern of my paper does not lie with pragmatism, in an attempt to avoid addressing and defending any one of the many varieties of pragmatism, I choose the word "simple" to describe the pragmatism I have in mind. As far as I am aware, none of the varieties of pragmatism are called simple pragmatism. I am grateful to an anonymous referee for pointing out that my simple pragmatism corresponds to Dewey's genuine pragmaticism.

10. Kitcher, "Mathematical Naturalism," 313, remarks: "Some stories, the stories of elementary mathematics, achieve the epistemic end of directly synthesizing the operations we find ourselves able to perform on physical objects (and on mental representations of such objects)."

11. See Alvin I. Goldman, "Reliabilism" in *A Companion to Epistemology*, ed. Jonathan Dancy and Ernest Sosa (Oxford: Blackwell, 1992), 433-36.

12. Dennis Lomas, "The Eye's Mind: a Philosophical Discourse on the Non-inferential and Conceptual Nature of Visual Perception and its Implications for Educational Theory" (Ph.D. diss., University of Toronto, 2000).

13. Wilfrid Sellars, *Empiricism and the Philosophy of Mind* (Cambridge: Harvard University Press, 1997).

14. John McDowell in "Having the World in View: Sellars, Kant, and Intentionality," *Journal of Philosophy* XCV (1998): 433, remarks: "Sellars sometimes suggests this helpful way of putting his thought: characterizations that affirm *epistemic* facts need to be distinguished from characterizations that affirm *natural* facts.…In these terms, his central thesis is that we must not suppose we can understand epistemic states or episodes in terms of the actualization of merely natural capacities — capacities that their subjects have at birth, or acquire in the course of merely animal maturation. I think 'epistemic' here amounts to something like 'concept involving'; I shall justify this interpretation shortly."

15. In vision science the role of the encoded knowledge base in inducing perception is spoken of in terms of "top-down" neuronal circuitry. See, for example, Patrick Cavanagh, "What's up in Top-down Processing?" in *Representations of Vision: Trends and Tacit Assumptions in Vision Research,* ed. A. Gorea (Cambridge: Cambridge University Press, 1991).

16. There are also ways to respond to perceptual inference theory by considering the basic structure of the perceptual neuronal system. For example vision scientist Zenon Pylyshyn in "Is Vision Continuous with Cognition? The Case for Cognitive Impenetrability of Visual Perception," *Brain and Behavior Science* 22, no. 3 (1999): 344, writes: "[P]erceptual principles, unlike the principles of inference, are responsive only to visually presented information." He contends that visual perceptual processing is "cognitively impenetrable." Pylsyshyn observes, 343, as I do above, that perceptual inference theory provides a fundamental pillar for the doctrine of theory ladenness of observation. His essay attacks perceptual inference theory across the board. In particular, Pylyshyn uses the argument about illusions, raised above.

17. Construing perception as both conceptual and non-inferential goes against much of current opinion in philosophy of mind. Generally inference is thought to be a necessary condition for concept application. There are exceptions. D.W. Hamlyn, for example, in "Unconscious Inference and Judgment in Perception," in *Perception, Learning, and the Self* (London: Routledge and Kegan Paul, 1983), 24, remarks: "[I]nstead of attempting to construe our perception of the world in terms of the ideas of data and inference or judgment, one might better invoke the notion of skill."